Errata of „Luck Logic and White Lies”

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Chapter 3, p. 19:

Win Class	Combinations	Probability
6 Correct	1	1/14 million
5 Correct + Bonus	6	1/2.3 million
5 Correct	252	1/55 491
4 Correct	13 545	1/1032
3 Correct + Bonus	17 220	1/812
3 Correct	229 600	1/61
loss (all the rest)	13 723 192	0.981

Because of the additional “super number” (a number chosen between 0 and 9), the highest winning category is subdivided into two classes, with the odds ratio 9:1. Thus the probability of landing in the top category (6 numbers out of 49 plus the super number) comes to only 1 in 140 million. In spite of increasing sales, not least due to the reunification of Germany, a winner in the highest category often does not appear for several weeks. The unclaimed cash is added to the jackpot for the next drawing. In 1994, the jackpot reached its highest value of about $24 million.

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p. 20:

A Game of Poker

In the game of poker, each player receives 5 cards from a pack of 52. Thus there are

$$\binom{52}{5} = \frac{52 \times 51 \times 50 \times 49 \times 48}{1 \times 2 \times 3 \times 4 \times 5} = 2\,598\,960$$

Chapter 5, p. 28:

As simple and plausible as the law of large numbers may seem, it is invoked incorrectly again and again. This is especially true in the situation in which an event is over- or underrepresented in a series of trials that is underway. How is it possible for such an imbalance to equalize, as predicted by the law of large numbers? It seems more likely that compensation in the opposing direction should have to take place. But is such compensation truly necessary? That is, after an excess of red numbers in roulette, can equalization take place only if thereafter, fewer red numbers appear than are to be expected based on the laws of probability? Let us take, for example, a number of series of 37 spins, each of which should yield, on average, according to the law of large numbers, 18 red numbers. But if in the first series, 25 red numbers appear, then red has exceeded the theoretical average by 7. If in the second series red appears 23 times, then the situation is even worse, since now the excess is $7 + 5 = 12$. A countervailing compensation has not taken place. Nevertheless, the relative frequency has moved in the direction of 18/37, namely, from 25/37 to $(25 + 23)/74 = 24/37$.

Chapter 6, pp. 35–36:

After three tosses, the numbers are 0.504 and 0.496, and after four tosses, 0.5008 and 0.4992. And what if the initial probabilities are something altogether different? After all, in the case of a real coin, the probabilities are unknown. In that case, we proceed from a general random experiment that can end with two possible results, which we can call "yes" and "no." If the associated probabilities are $(1+d)/2$ and $(1-d)/2$, then the (possibly negative) number d is a measure of the deviation from symmetry. That is, the smaller the absolute value of d, the less the experiment is likely to deviate from the symmetric ideal case. If two independent yes–no random decisions whose deviations from symmetry are given by d and e are made one after the other, this leads to the following probabilities:

- two yes or two no:

$$\frac{(1+d)(1+e)}{4} + \frac{(1-d)(1-e)}{4} = \frac{1+de}{2};$$

- one yes and one no:

$$\frac{(1+d)(1-e)}{4} + \frac{(1-d)(1+e)}{4} = \frac{1-de}{2}.$$

Thus the measure of deviation from symmetry in the total experiment is equal to the product $d \times e$ of the individual measures d and e. In our example of the bent coin with probabilities $0.6 = (1+0.2)/2$ and 0.4, this measure was 0.2. Therefore, in multiple flips of the coin, the probability of a yes decision is

- $(1+0.2^2)/2 = 0.52$ for two tosses,
- $(1+0.2^3)/2 = 0.504$ for three tosses,
- $(1+0.2^4)/2 = 0.5008$ for four tosses,
- $(1+0.2^5)/2 = 0.50016$ for five tosses,

and so on.

Thus even with such an unfair coin, a fair, equiprobable decision can be made. The only condition is that each of heads and tails can appear. Thus in a $90:10$ situation, which means that $d = 0.8$, after 20 tosses, the probabilities are 0.5058 and 0.4942.

We shall not consider the more complicated case of a die in great detail. However, the result is largely analogous. We begin with a die whose basic probabilities are $p_1, p_2, \ldots, p_6$, all in the range from $(1-d)/6$ to $(1+d)/6$, where d is a number in the range from 0 to 1. In order to obtain a result among the numbers $1, 2 \ldots, 6$, one could write down the six numbers in a circle, and move a counter around the circle the number of steps equal to the number that appears on the die. One obtains the same result by considering only the remainder upon dividing the sum of the numbers that appear on the die by six. The longer one tosses the die, the more the probabilities of the different results approach one another. It can be shown[3] that after n throws, the probabilities for the six individual fields all lie in the range from $(1-d^n)/6$ to $(1+d^n)/6$. As in the case of the coin, this leads slowly but surely to an equal distribution (up to a small error) of the probabilities. An asymmetry in the die has thus been overcome. In contrast to the case of the coin, the six basic probabilities cannot be too large. If a die is so loaded that one event has a probability of $1/3$ or greater, then it is no longer certain that the procedure described will lead to the advertised result.

[3] One can proceed as in the case of a coin. That is, one considers two random experiments with the results $1, 2, \ldots, 6$. If the probabilities of a random decision lie between $(1-d)/6$ and $(1+d)/6$, and that of a second decision between $(1-e)/6$ and $(1+e)/6$, then the probabilities of the combined experiment lie between $(1-de)/6$ and $(1+de)/6$. In the calculations that follow, one represents the probabilities in the form $(1+d_1)/6, \ldots, (1+d_6)/6$, with

$$d_1 + \cdots + d_6 = 0 \quad \text{and} \quad |d_1|, \ldots, |d_6| \le d$$

(and analogously for the second experiment).

Chapter 9, p. 55:

Original choice is **not** changed:

First Choice Was	Probability	Conditional Probability of Winning	Net Probability
Correct	1/3	1	1/3
Incorrect	2/3	0	0
Total Probability of Winning (Sum)			1/3

Original choice is ~~not~~ changed:

First Choice Was	Probability	Conditional Probability of Winning	Net Probability
Correct	1/3	0	0
Incorrect	2/3	1	2/3
Total Probability of Winning (Sum)			2/3

Chapter 11, p. 69:

One might now ask whether the advantage to the second player increases if dice with different numbers are used. Let us formulate the question in greater generality. We seek independent random variables $X_1, X_2, \ldots, X_n$ for which the minimum of the probabilities $\mathrm{P}(X_1 > X_2)$, $\mathrm{P}(X_2 > X_3)$, $\ldots$, $\mathrm{P}(X_n > X_1)$ is as large as possible. For the case $n = 3$ random variables the maximum turns out to be 0.618, where the random variables satisfying that condition cannot be achieved with suitably numbered dice. Moreover, it is clear that the value $21/36 = 0.583$ determined for the game that we have been considering cannot be much improved. In contrast, for the case $n = 4$ independent random variables, the theoretical maximum $2/3$ can actually be achieved with real dice:[1]

Chapter 12, p. 73:

Let us see what that last statement means for our series of dice throws. For one throw, we have an expectation of 3.5 and a standard deviation of 1.708. For the random variable X defined as average of 10 000 diced numbers, we obtain the standard deviation $\sigma_X = 0.01708$. According to Chebyshev's inequality, for 10 000 throws, the average of the numbers thrown should lie outside the interval $3.5 \pm 10\sigma_X$ with probability at most 0.01. And it is precisely this unlikely event that has occurred in our series of trials with the result 3.7241. If we are looking for a fair die, then this one should not be used, since we are forced to reject the hypothesis of symmetry.

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p. 74:

Now that we have seen how useful Chebyshev's inequality can be, we would like to delve into its consequences a bit more deeply. In mathematical formulation, for a random variable X, we have

$$\mathrm{P}(|X - \mathrm{E}(X)| > t\sigma_X) \leq \frac{1}{t^2}.$$

Chapter 15, p. 101:

The theoretical considerations and the not-so-simple calculations can be avoided if one uses a Monte Carlo method. That is, before one actually carries out a series of trials with a die that is under investigation, one should simulate a large number of series of trials on the computer. For example, one can run 999 series of trials. If the result with the die is significantly different in comparison to these 999 results, then the hypothesis that the die is symmetric can be rejected. But what is meant by "significantly different"? Since asymmetric dice produce larger χ^2 values than symmetric dice, one simply considers the greatest ten of the 999 simulation results as outliers. If the test result lies above those of the 989 lowest simulation results, then there are two possible causes:

- the die is asymmetric; the hypothesis is correctly rejected.
- the die is symmetric, so that the result of the experiment is an outlier; rejecting the hypothesis in this case is an error.

Chapter 16, p. 118:

In particular, using the limiting value of A^n, which in the ruin problem we obtained by other means, one can determine which absorbing states will be reached from a particular start state with what probabilities. How long this will take on average can be determined from the matrix $(I-Q)^{-1}$. If ℓ is the column vector whose coordinates equal the expected number of steps until an absorbing state is reached, then just as in the ruin problem, one obtains the equality

$$\ell = Q\ell + \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix},$$

which can be transformed into

$$\ell = (I-Q)^{-1} \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}.$$ ["+" was removed]

Chapter 17, p.122:

more. If an ace is drawn, it must be counted as 11 unless the result would be in excess of 21. Once the bank has finished drawing, each player who has not gone over 21 computes his or her score. A player whose total exceeds that of the bank gets his bet plus the amount of his bet. If the win occurs with a blackjack, then he gets his bet plus one and one-half times his bet. If it is a tie, the player gets back his bet. If the bank has the better hand, the bet is lost.

In contrast to roulette, in blackjack, the players have considerable strategic influence over the course of the game in deciding whether to take additional cards. To make the game more interesting, there are several additional rules.

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p. 139:

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\begin{aligned}
\mathrm{P}(2) = \cdots = \mathrm{P}(6) &= \frac{1}{13} - \frac{C}{10n}, \\
\mathrm{P}(10) = \cdots = \mathrm{P}(\mathrm{Ace}) &= \frac{1}{13} + \frac{\mathrm{C}}{10\mathrm{n}}, \\
\mathrm{P}(7) = \mathrm{P}(8) = \mathrm{P}(9) &= \frac{1}{13}.
\end{aligned}
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Chapter 42, p 447:

42

Baccarat: Draw from a Five?

Should a baccarat player whose first two cards total five request another card?

The game of baccarat—also known as chemin-de-fer—with its over 500 year history, is the most widespread casino card game after blackjack.[1] As with blackjack, the game is usually played with several decks of cards. To win, a player must draw a higher hand than that of the bank; equal hands result in a draw. The values of the cards are as follows: the ace has value 1, cards 2 through 9 have their face values. Face cards and 10 count zero. The card values are summed modulo ten. Thus an 8 and a 6 total 4, while a jack and an ace total 1.

A game of baccarat begins with the player and banker each receiving two cards face down, which are not revealed to the other player. If either player has a hand worth 8 or 9, then both ~~players~~ show their hands and the game is scored. Otherwise, the player decides whether he wishes to have an additional card dealt. If he chooses to receive a card, it is dealt face up. Then it is the banker's turn. He, too, is allowed to receive a third card, where in making his decision he can take into account his own hand, the decision of the player, and the exposed card if such exists. At this point the game is over; banker and player reveal their hands, and the game is scored.