

# Skill and Chance in Games

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# Curriculum vitae



1975-1982

Study of mathematics at the University of Bonn

1985

Ph.D. (Bonn): Number theory, algebraic topology

1985-

R&D of slots and other coin op machines

1998-2019

General manager of subsidiaries of Gauselmann AG

1998

Textbooks on

- Mathematics of games, US: 2005

2002

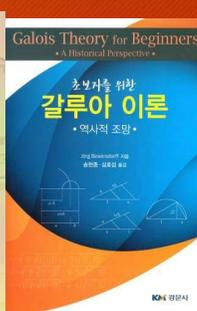
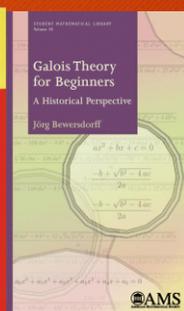
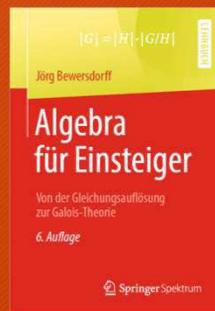
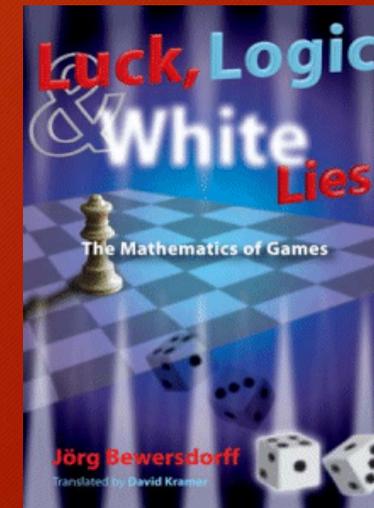
- Algebra (Galois theory), US: 2006, Korea: 2015

2011

- Mathematical statistics

2014

- JavaScript and object-orientated programming



## Skill vs. chance: Why does it matter?

Gambling, i.e. playing

1. a game of chance with
  2. a monetary (or valuable) stake and
  3. a chance to win a an amount (or a valuable price),
- is strictly regulated in most jurisdictions  
(and sometimes absolutely prohibited).

## Section 284 Criminal code (D, § 284 StGB)

(1) Whoever, without the permission of a public authority, publicly organizes or runs a game of chance or makes the equipment therefore available, shall be punished with imprisonment for not more than two years or a fine.

(2,3,4) [...]

Section 285:

Whoever participates in a public game of chance (Section 284) shall be punished with imprisonment for not more than six months or a fine of not more than 180 daily rates.

# From gambling as moral issue ...



San Zeno altarpiece in Verona

(painter: Andrea Mantegna, 1431-1506)

Soldiers dived Jesus' garment

(the Gospel of John, ch. 19, verse 24, tells that they casted lots)

## ... to consumer protection:

- The critical view on gambling changed its motivation: From earning money without working as moral problem to consumer protection.
- Consumers, i.e. players and especially pathological gamblers, must be protected!
- In this context gambling is comparable to other problematic products like alcohol, tobacco, sugar and fat fast food.
- Illegal gambling cannot be controlled. Therefore activities of gambling should be channeled to attractive, legal and controlled alternatives.
- Are skill games less dangerous? Or only less immoral because there is not any win without “working”?

# Classification of games in the past ...



*Libro de los juegos* (book of games),  
commissioned by Alfonso X of Castile, finished  
1283:

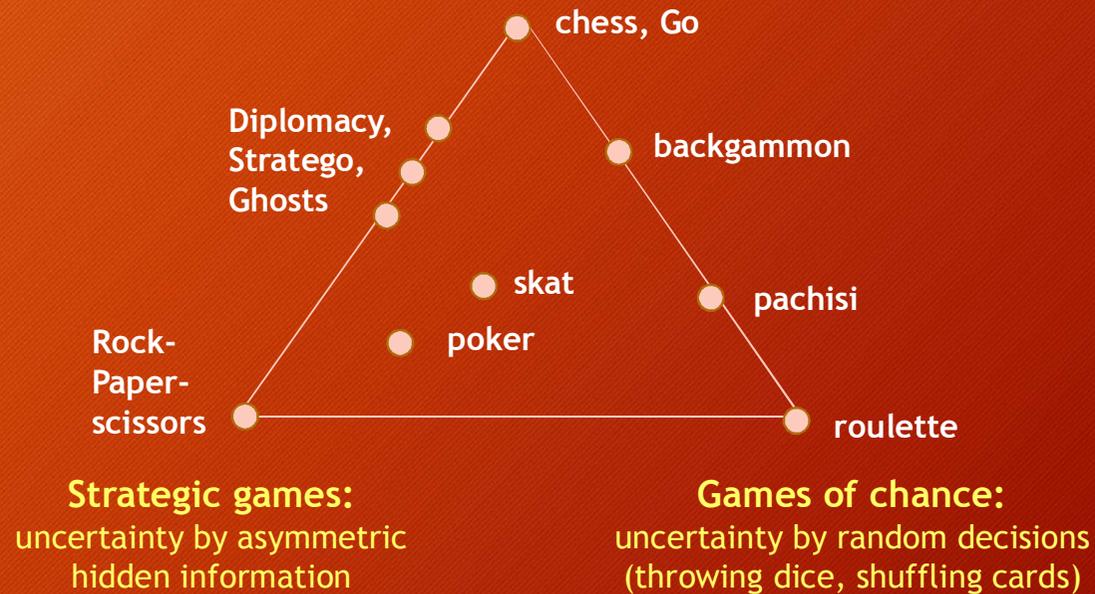
The book explains the difference between

- a game of pure, intellectual strategy (chess),
- a game of pure chance (dice) and
- a game that incorporate both elements (backgammon).

# ... and today: Where uncertainty comes from? \*

(\*) not considered:  
games with importance of  
• manual skill (e.g. Mikado) or  
• speed of response (e.g. blitz chess)

**Combinatorial games:**  
uncertainty by a huge number of combinations of options in a move sequence



**Diplomacy:** simultaneous moves

**Stratego:** hidden type of pieces

**Ghosts:** hidden type of pieces

# My talk: An outline

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Options and limits of mathematical methods
- III. Measuring chance and skill  
The importance of symmetry
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Methods of reduction to symmetric games
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# I. It's a legal question!

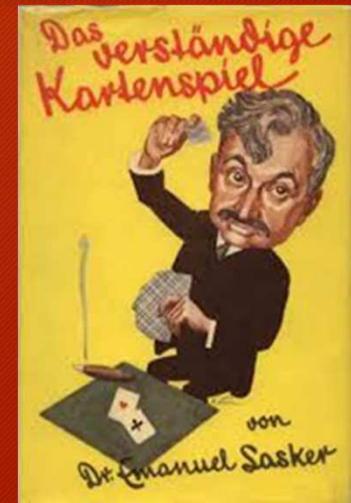
The distinction of games of chance and games of skill

- is based on national legislation and legal interpretation in judgements, commentaries and other legal literature.
- Differences are possible, also between the member states of the European Community.
- In particular the German jurisprudence (and not only it) reveals a skepticism towards purely mathematical considerations, ...

... for example

“Mathematical and complicated probabilistic calculations as described in Dr. Lasker’s book ‘Das verständige Kartenspiel’ [intelligent card game], which was presented by the plaintiff, must be rejected for the decision of the judgement. The decision whether a game has the character of a game of chance, must be based on the **average ability** of the players.”

German Federal Administrative Court, Judgement on 17 May 1955, case number: BVerwG I C 133.53



Emanuel Lasker (1868-1941),  
mathematician,  
Chess Champion for 27 years.  
The book was published in 1929,  
similar to “Encyclopedia of games“, 1929.

# Germany: § 284 criminal code (StGB), ...

„A game of chance ... is a game in which ... the decision to win or lose does not depend dominantly on the abilities, knowledge and level of attention of the players, but solely or dominantly on chance, i. e. on the action of unpredictable causes, deprived of the influence of persons involved.“

Imperial Court of Justice (Reichsgericht), judgment on 18 May 1928, case number: I 977/27

48. 1. Unter welchen Voraussetzungen kann ein Spiel an Geldspielautomaten, insbesondere an sog. Bajazzoapparaten, als Glücksspiel beurteilt werden?

2. Zum inneren Tatbestand des § 284 StGB.

3. Unter welchen Voraussetzungen kann Tateinheit zwischen den Vergehen nach den §§ 284, 284a und 285 StGB. angenommen werden?

I. Straffenat. Urt. v. 18. Mai 1928 g. R. I 977/27.

- I. Schöffengericht Leipzig.
- II. Landgericht daselbst.

I. 1. Ein Glücksspiel im Sinne der von den Vorderrichtern angewandten Strafvorschriften ist ein Spiel, bei dem nach den Vertragsbedingungen, die von dem einen Teil ausdrücklich oder durch schlüssige Handlung angeboten und vom anderen Teil angenommen werden, die Entscheidung über Gewinn oder Verlust nicht wesentlich von den Fähigkeiten und Kenntnissen sowie vom Grade der Aufmerksamkeit der Spieler, sondern allein oder hauptsächlich vom Zufall, d. h. vom Wirken unberechenbarer, dem Einfluß der Beteiligten entzogener Ursachen abhängt. Soweit zur Erfüllung eines Tat-



Subject of the trial was the "Bajazzo" game

## ... Interstate Treaty on Gambling (GlüStV), ...

“A game of chance exists if, in the context of a game, a fee is required for acquiring a chance to win and the determination of winnings is entirely or predominantly a matter of chance. In any case, the determination of winnings is a matter of chance if, in this regard, the uncertain occurrence or outcome of future events is decisive. Games of chance also include wagers against payment on the occurrence or outcome of a future event.”

Section 3 (1), Interstate Treaty on Gambling, signed 15 December 2011

## ... Gaming Order (SpielV), ...

“Anyway, the mentioned card games [Bridge, Skat, Doppelkopf and others] require a certain time period to be considered as a game of skill. They become a game of chance if the random composition of the cards is not neutralized by a longer time of playing, giving each player roughly the same starting point on which to develop his skill.”

Comment on section 5a of Gaming Order: *Landmann/Rohmer*, *Gewerbeordnung und ergänzende Vorschriften*, Vol. II, § 5 a SpielV, margin number 8 (Juni 2015, editor: Marcks)

## ... Section 33 d Trade Regulation (GewO)

- “... the Appeals Court confused two things: the need to **determine the nature of the game with scientific methods** and the scientific methodology as part of the use of skill which must be disregarded because of the excessive demands of the average player.“
- “... the hearing of evidence was wrong ... because ... all players were trying to win using their potential of skill instead of—as it would have been necessary—that one player acted randomly in the games of the played sequence.“

Federal Administrative Court (BVerwG), judgement of 9 October 1984 – case number: C 20.82

## But it can also be bizarre ...

“Playing chess against a computer becomes—in spite of the rules of the game dominated by logic—a game of chance, if the conditions are set in such a way that the computer can use its superiority based in the program and therefore the average player has no chance, also playing with maximal effort.”

Administrative Court Wiesbaden, judgement on 10 October 1995 (case number: 5/3E 32/94, Gewerbearchiv, 1996, pp. 68-69)

The judgement was approved by the Federal Administrative Court, but without this general “outlook”.



Subject of the trial was a claw machine

... and it can also be detailed and professional



## Liechtensteinisches Landesgesetzblatt

Jahrgang 2010

Nr. 441

ausgegeben am 29. Dezember 2010

### Verordnung

vom 21. Dezember 2010

## über Geschicklichkeits-Geldspiele (GGV)

### Art. 2

#### *Abgrenzung zwischen Geschicklichkeits-Geldspielen und Glücksspielen*

1) Ein Geschicklichkeits-Geldspiel liegt vor, wenn der in Aussicht gestellte Gewinn in unverkennbarer Weise ganz oder überwiegend von der Geschicklichkeit des Spielers abhängt, wobei auf die Fähigkeiten und Fertigkeiten eines durchschnittlichen Spielers abzustellen ist.

2) Ein Geschicklichkeits-Geldspiel liegt insbesondere dann nicht vor, wenn:

- der Spieler bei Durchführung einer Mehrzahl von Spielen einen Gewinn erzielen kann, ohne dass er Einfluss auf den Spielverlauf nimmt;
- der überdurchschnittlich fähige Spieler bei Durchführung einer Mehrzahl von Spielen keinen angemessen höheren Gewinn erzielen kann als der durchschnittlich fähige Spieler;

c) die Auszahlquote bei Durchführung einer sehr grossen Anzahl Spiele vorgegeben ist und unter 100 % liegt. Dabei bleiben im Falle von Spielturnieren "Spieler gegen Spieler" die vom Veranstalter erhobenen Kommissionen für die Organisation oder Durchführung des Spiels ("rake") für die Berechnung der Einsätze unbeachtlich;

d) die Spieldurchführung nicht transparent ist; oder

e) dem Spiel kein von der Gewinnmöglichkeit unabhängiger Unterhaltungswert zukommt.

3) Als Geschicklichkeitsspiele im Sinne dieses Artikels gelten insbesondere:

a) Schach;

b) Bridge;

c) Backgammon;

d) Schieber und ähnlich anforderungsreiche Formen des Jassens;

e) Quizspiele um Wissensfragen;

f) Spiele mit Sportcharakter wie Billard oder Dart.

4) Alle Formen von Black Jack und Poker gelten als Glücksspiele.

5) Bestehen Zweifel, ob ein Geldspiel als Geschicklichkeits- oder als Glücksspiel zu qualifizieren ist, so kann das Amt für Volkswirtschaft von sich aus oder auf Gesuch einen Entscheid fällen. Dabei berücksichtigt es auch, ob sich das Spiel zum Glücksspiel eignet oder leicht zum Glücksspiel abändern lässt.

# Honored by the Austrian Federal Fiscal Court



UNABHÄNGIGER  
FINANZSENAT

Außenstelle Wien  
Senat 21

GZ. RV/1662-W/06

## Berufungsentscheidung

Der unabhängige Finanzsenat hat über die Berufungen des Dr. Wolfgang Leitner als Masseverwalter der Bw., 1010 Wien, Kohlmarkt 14, gegen den Bescheid gemäß § 201 BAO des Finanzamtes für Gebühren und Verkehrsteuern Wien vom 19. Mai 1994 betreffend Rechtsgebühren gemäß § 33 TP 17 GebG und Erhöhung für den Zeitraum 1. Februar 1994 bis 28. Februar 1994 entschieden:

1.) Der Berufung wird teilweise Folge gegeben und der angefochtene Bescheid abgeändert:

Die Rechtsgebühren  
Euro 3,133.120,

Im Übrigen wird

2.) Der Berufung  
Erhöhung von E

(Schillinginform:  
S 43,112.675,0€

### IV.) Verwendete Literatur:

- Glücksverträge – Gewagte Geschäfte, online Lehrbuch Zivilrecht, Kapite Barta: Zivilrecht – Grundriss und Einführung in das Rechtsdenken (<http://www.wiley.com>)
- Jörg Bewersdorff, Glück, Logik und Bluff, Mathematik im Spiel – Methos Auflage, vieweg
- Gerhard Strejcek/Dietmar Hoscher/Markus Eder, Glücksspiel in der EU i
- Dr. Rudolf Sieghart, Die öffentlichen Glücksspiele, Wien, 1899
- Erlacher, Glücksspielgesetz, Stand 1. Oktober 1997, 2. Auflage, Verlag
- Schwartz-Wohlfahrt, Glücksspielgesetz mit wichtigen Spielbedingungen
- Frank Höpfl, Zum Beweissthema der Abhängigkeit eines Spieles vom Zuf Mathematik 1978,
- Christian Berti, Klaus Schweighofer, Österreichisches Strafrecht, Beson
- Ürek Vedat, Das Glücksspielstrafrecht und die Pokercasinos in Österreik

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können entscheiden, ob sie Karten zukaufen wollen oder nicht. Die zugekauften Karten dürften offen vor den Spieler hingelegt worden sein. Die Karten des Spieler-Gebers werden dann aufgedeckt und er kann dann 1 Karte kaufen. Wenn der Spieler-Geber mit dem Kaufen der Karten fertig ist, werden die Karten der Gegenspieler der Reihe nach aufgedeckt und gegen das Blatt des Spieler-Gebers verglichen. Wer näher an der Punktezahl 9 ist, der Spieler oder der Spieler-Geber, der hat gewonnen. Auf die Kartenkombinationen der anderen Spieler hat das keinen Einfluss. Zum Beispiel Spieler-Geber und Spieler tätigen je einen Einsatz von 50, dann bekommt der, der gewonnen hat, 100. Jeder einzelne Spieler hatte ca. 5 Schilling pro Spielbox zu zahlen.

"Bluff" wie beim Pokerspiel kann es hier durchaus geben, allerdings nur zwischen Spieler und Spielergeber. Das resultiert aus den verdeckten Karten.

Ein Taktieren ist im Hinblick auf die Merkfähigkeit der Kartenkombinationen aller am Spiel beteiligten Spieler möglich – wie beim Black Jack, allerdings hier bezogen auf mehrere Spielrunden.

Nach Auskunft des Geschäftsführers gab es bei Lucky 9 Turnierspiele. Diese Turnierspiele wurden wie beim Poker nach einer eigenen Spielregel durchgeführt.

### 7.1. Spielstrategische Momente

Laut Bewersdorff, Glück, Logik und Bluff, 301 ff, Kapitel: Baccarat: Ziehen bei Fünf?: „Sehen wir uns die Spielchancen zunächst auf rein intuitiven Niveau an: Spieler und Bank haben nur dann Entscheidungen zu treffen, wenn beide Ausgangsblätter einen Wert von 0-7 ergeben. Um ein möglichst günstiges Blatt zu erhalten tun Spieler und Bank gut daran, bei niedrigen Werten eine dritte Karte zu ziehen; dagegen kann bei Werten von 7 oder knapp darunter meist auf eine Karte verzichtet werden. Speziell der Spieler muss allerdings bedenken, dass er mit seiner Entscheidung der Bank einen Hinweis auf die mutmaßliche Qualität seines Ausgangsblattes gibt. Da eine dritte Karte offen ausgeteilt wird, lassen sich diese Hinweise, wenn auch in Grenzen, gegebenenfalls sogar auf das Gesamtblatt übertragen. Insgesamt kann die Bank damit ihre Strategie immer dann erfolgreich anpassen, wenn die Handlungen des Spielers Rückschlüsse auf dessen Ausgangsblatt zulassen.“

Im Hinblick auf die Merkfähigkeit der Kartenkombinationen wurde gleichzeitig wie zu Black Jack eine so genannte Zählsystem mathematisch entwickelt, um sich einen ungefähren, aber ausreichenden Überblick über die Kartenzusammensetzung des Kartenstapels zu verschaffen.

### 8.) Die Grundstruktur von Concord Aces

Von der Bw. wurde das Kartenspiel Concord Aces angeboten. Concord Aces gilt dem Black Jack ähnlich. Laut [http://de.wikipedia.org/wiki/Black\\_Jack](http://de.wikipedia.org/wiki/Black_Jack) ist Black Jack das meistgespielte

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aufgedeckte Karte. Hat der Spieler einen Black Jack (bestimmter Kartenwert) hat er das Spiel gewonnen. Er erhält seinen Einsatz zurück und darüber hinaus eine Gewinnauszahlung in Höhe des Eineinhalbfachen seines Einsatzes: zB 100 Euro Einsatz + 150 Euro Gewinn. Wenn die Bank jedoch ebenfalls mit einer 2. Karte einen Black Jack erreichen könnte, erhält der Spieler zunächst keine Auszahlung.

Die Autoren berechnen die bedingten Wahrscheinlichkeiten der Punktestände der Bank, mit denen sie ihr Spiel beendet, bzw. des Spielers um das Spiel zu gewinnen bzw. zu verlieren und kommen zu folgendem Schluss: Die Gewinnererwartung ist nur dann positiv, wenn bestimmte Spielregeln gelten und sofern sich der Spieler an bestimmte Spielregeln hält. Freilich ist die Gewinnererwartung unter den erwähnten Spielregeln nur unwesentlich größer als Null – sie liegt ungefähr bei einem Prozent. Black Jack ist das einzige bekannte Glücksspiel mit unter bestimmten Bedingungen positiver Gewinnererwartung für den Spieler. Black Jack ist im Vergleich zum Roulette das für den Spieler günstigere Spiel. Die „ultimative“ Black Jack Strategie hat emotional im Grunde wenig Prickelndes zu bieten: Die Bank muss sich wie ein Automat verhalten, und der Spieler sollte sich wie ein Automat verhalten, wenn er nicht verlieren will.“

Laut Bewersdorff, Glück, Logik und Bluff, 81 ff, Kapitel: Black Jack, Ein Märchen aus Las Vegas, haben die Spieler bei Black Jack, anders als beim Roulette einen erheblichen strategischen Einfluss, da sie entscheiden, ob sie noch weitere Karten ziehen wollen oder nicht. In der Literatur zu Black Jack wurde eine Strategie beschrieben, wonach ein Spieler ca. 3,3% bis 10% Gewinnchancen hat. Grundidee war es sich mittels eines so genannten Zählsystems einen ungefähren, aber ausreichenden Überblick über die Kartenzusammensetzung des Kartenstapels zu verschaffen:

„Diese Zählsystem erfordern allerdings ein Höchstmaß an Übung und Konzentration, denn jede im schnellen Spielverlauf getroffene Fehlentscheidung geht im Mittel zu Lasten des Spielers. Nur wer ständig richtig zählt und seine Strategie entsprechend anpasst, kann seinen geringfügigen Vorteil gegen die Bank halten. Erfolgreiche Card-Counter dürften daher in der Masse der alles andere als optimal spielenden Durchschnittsspieler untergehen. Dass sich der minimale Vorteil zudem nur auf die Erwartung bezieht und durch Pech im Einzelfall zunichte gemacht werden kann, versteht sich von selbst.“

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## II. Mathematics: To get logic and transparent

- Mathematics as a meta-science should be able to formally characterize any consistent system of rules:
  - Mathematics does not include any definition of a “point” (in geometry), “speed” (in physics) or “skill game” resp. “predomination of skill” (in legal applications).
  - ✓ Mathematics describes relations between objects (“attributes” on the level of an application are objects, too).
  - ✓ Only the logical implications are mathematized (and therefore they become more precise).
- We use methods which are analogous to physics, where simple experimental setups and thought experiments are investigated: Therefore we will design artificial games using a “construction set”. This approach allows a transparent verification of compatibility of mathematical and legal view.
- The approach should be variable concerning the types of players which are defined as legally relevant:
  - (better, worse) average player / beginner / random player,
  - organizer / dealer / machine: To be viewed as a fictitious player?

# Mathematical “construction set” for games

Methods to construct new games based on known games:

- Random decision who is to move first.
- Random decision on whether another game is played instead of the original game, especially in the form of pure game of chance or a pure game of skill.
- Sequence of repeated games with summed up payoffs (“cash games”) or another scoring (e.g. the winner takes it all or tournament consisting of knockout games).
- Sequence of several games with permuted roles (who moves first, second, ...) to get a symmetric game.

Using these constructions we get some instructive examples of games ...

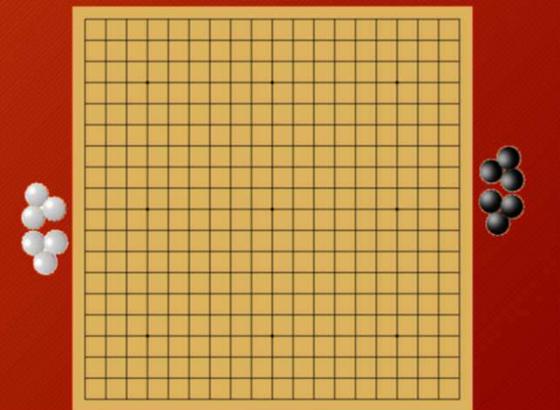
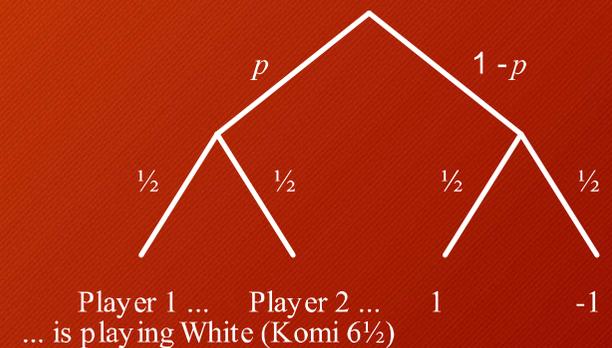
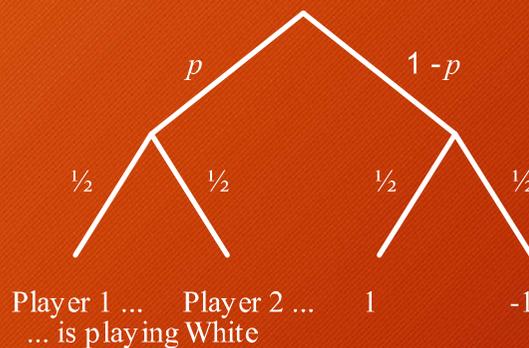
# 1. “Topology” of games:

- Take a fixed probability  $p$ .
- Modify the rules of chess by adding a random decision before each move: With probability  $1 - p$  the move is replaced by a random decision which selects randomly the move (uniformly distributed).
  - Using  $p = 0$  we get a pure game of chance.
  - Using  $p = 1$  we get original chess.
  - The change by varying  $p$  is continuous. Where is the transition between games of chance and games of skill? There is no evidence for a transition at  $p = \frac{1}{2}$ .

## 2. Game theory & complexity: two variants ...

- Take a fixed probability  $p$ .
- With probability  $1 - p$  the winner is selected randomly (equally likely), otherwise the colors of the following game are decided randomly:

1<sup>st</sup> variant: end game of chess: king and rook (W) versus king (S),  
2<sup>nd</sup> variant: Go with komi 6½ (no draw is possible).



## ... and how they are played:

- All games (both variants, all  $p$ ) are symmetric.
- $p = 0$  is corresponding to a pure game of chance.
- $p = 1$  is corresponding to Go resp. the chess endgame (colors equally likely)
- In the view of game theory (minimax) there no difference between both variants (fixed  $p$ ).
- But a champion will win with probability  $1/2 + p/2$  the 2<sup>nd</sup> variant (based on Go) and with probability  $1/2$  the 1<sup>st</sup> variant (pure chance).
- Go with  $p = 1/2$  is a game with “50 % skill”: a champion will win 75 % of the played games.

### 3. Sequence of repeated games

- Fix a probability  $p$ .
- Based on this value  $p$  the Go based variant form of section 2 is repeated  $n$  times.
- The payoffs are summed up.
- How much repetitions  $n$  are needed in relation to the probability  $p$  (e.g.  $p = 1/4$ ) that the whole sequence of games is predominated by skill?

## 4. Different amounts as payoffs

- First an amount is fixed randomly or by bids by the player. This amount will be the payoff for the winner:
- After this the Go variant of section 2 is played.
  - $p = 0$  corresponds to a pure game of chance.
  - $p = 1$  corresponds to a pure game of skill because a champion will win each game.
  - $p = \frac{1}{2}$  corresponds to a game of a 50:50 mixture of chance and skill.

## 5. Rock-paper-scissors

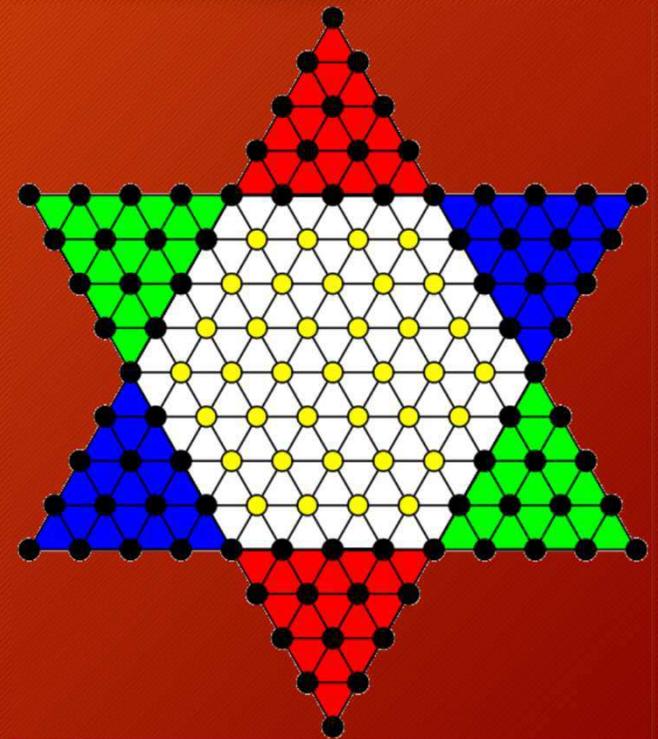
- Outcomes are only determined by decisions of the player,
- No influence by random decisions.
- The game is symmetric and
- Equivalent to a bet:  
One player is making his hidden selection by choosing a card, hidden to his opponent. Then e.g. the opponent's decision "rock" is equivalent to two bets:
  - ✓ Bet half of the amount on the event that the first player has selected "scissors" and
  - ✓ half of the amount that the first player has *not* selected "paper".
- Intellectual abilities such as good memory, quick reaction and attention have no meaning, therefore:
- There should not be significantly different levels of competence.

## 6. Tournament with multiple players

- Tournament with  $2^n$  players with single-elimination (knockout).
- During the tournament the Go variant from section 2 with  $p = 0,501$  is played.
- The tournament is predominated by skill.
- A champion will win with probability approximately equal to  $0,75^n$ , e.g. 0,178 in the case of 64 players.

## 7. A game of pure skill with multiple players

- Chinese checkers with  $n = 3$  to 6 players.
- Play  $n!$  (= 6, 24, 120 resp. 720) games with all permutations of players concerning first moving, ...
  - The game is symmetrical.
  - There is not any random decision. Therefore the game is usually classified as a game of pure skill.
  - Nevertheless, no player can force on his own a total result which is at least balanced.



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### III. Demarcation of skill and chance

- Does an event (and its probability) exist that a particular game is won by skill? What is therefore necessary is a characterization of each move sequence as skill or as chance. This may work for an easy game like in section II.2. But what about a “real” game like backgammon???
- Only payoffs can be observed definitely. Their frequency can be counted in a sequence of games.
- Thus, player’s influence can be rated absolutely—and in consequence also relative to other options of influence (i.e. strategies).
- Already in 1928 the Imperial Court of Justice compared the success of skillful and “blindly”/randomly acting players ...
- In symmetric games: Comparing players is equivalent to comparing strategies ...

# Demarcation of skill & chance under symmetry

- Subject of the “Bajazzo judgement“ from 1928 was the **asymmetric** case “player against machine“

In a legal sense “Bajazzo” is a game of two players. One of them is inactive. But he wins often. And he does not use skill.

The court commissioned experts for making statistical tests. They compared the results of a blindly/randomly acting player (9 to 24%) with the results of a normal player (31 to 36 %). The maximal difference  $36 - 9 = 25 \%$  is smaller than the probability that the machine wins (without using any skill!).

- In **symmetrical** games the influence of skill can be measured with a simple comparison of both players. For example:

If the better player wins 76 % of the games of a sequence and the worse player only 24 %, then the difference of 52 % is a consequence of the different competence levels (if the sequence is long enough to minimize statistical deviation).



# The criterion in detail

- The comparison of success in symmetric games results is a **sufficient but not necessary condition**:
  - Of course, two players of nearly the same competence level will not produce a strong unbalanced result.
  - But it works: For the variant “chess end game” from section II.2 there will be no experienced players who will get a strong unbalanced result.
- It seems that the criterion can also be referenced to one game. In this version the difference of relative frequencies has to be interpreted as “probability” (of the “event” that the game is decided by skill):
  - In the case that the complementary “probability” (= “probability” of winning the game by chance) exceeds  $\frac{1}{2}$ , then each of both players wins with probability of  $\frac{1}{4}$  or more.

# My talk: An outline

## Introduction

- I. It's a legal question!  
Legal principles in Germany
- II. Mathematics: To get logic and transparent  
Options and limits of mathematical methods
- III. Measuring chance and skill  
The importance of symmetry
- IV. General (non symmetric) games  
Methods of reduction to symmetric games
- V. Conclusions

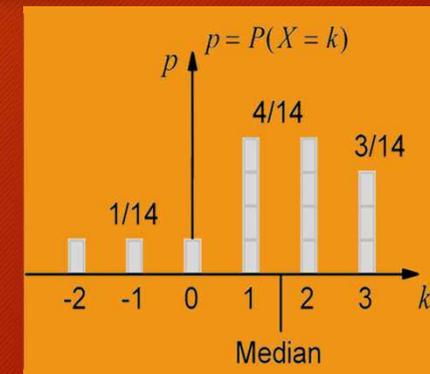
## IV. General games

- We are looking for generalizations of the criterion. We are looking for games
  - which are not symmetric or
  - which include some payoffs other than +1 and -1 or
  - which are played with more than two players.
- Dependent of the type of a game different approaches are needed to get a sufficient criterion for a predomination of skill.
- But: You should never simply talk about „Poker“. An analysis is only possible for a precisely defined variant, e.g. “Texas Hold‘em”, with a fixed number of players, a fixed number of rounds of betting, fixed limits and, if applicable, with fixings of length and mode of the tournament.

# 1. Games with payoffs not only = $\pm 1$

Given is symmetric game for two players:

- We start with an empirical measured probability distribution of payoffs which a good player obtains against a worse player.
- We get a sufficient condition for a predomination of skill: lead the assumption that maximal 50 % of the games are decided by skill to a contradiction
  - Remove the best 50 % of the payoffs. Because of the assumption the rest of payoffs must be a result of random influence. If there is any advantage for the better player (e.g. conditional mean value  $> 0$ ) we get a contradiction to the symmetry (see diagram).
  - In the case of a normal distributed payoff  $X$  of the better player:  $E(X) > \sqrt{\frac{2}{\pi}} \sigma_X$
  - In the case of sequence of  $n$  repeated games ( $Y =$  payoff of a single game):  $E(Y) > \sqrt{\frac{2}{n\pi}} \sigma_Y$



## 2. Non symmetric two-person games

- The rules are symmetrized:
  - Play two games with permuted roles, e.g. colors (e.g. “White” moves first).
  - Play one game with randomly decided colors.
- Play a sequence of symmetrized games:
  - *Example:* If the better player wins 80 % of the games playing as White and 72 % of the games playing as Black, then the sufficient criterion is fulfilled:  
The better player wins 76 % of the symmetrized games .

### 3. More-person games: Predominated by skill?

- Try to reduce the game to a two-person game.
- Dependant of the type of game there are different possibilities (because of the sufficient character of the criterion there is no contradiction between them):
  - Games without interaction and with highscore (like bowl):  
It is enough to compare pairs of two players.
  - Tournaments consisting of two-person games in knockout mode:  
It is enough to focus the single two-person games.
  - Games which can be played with variable number of players starting form two:  
The character of the two-person variant can be seen as hint for the character of the more-person game.
  - Other more-person games:  
The formation of coalitions usually changes the character of a game. However, the influence and information of the players do not change. Therefore look for games with two coalitions.

# My talk: An outline

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## V. Conclusions

- Mathematics does not include any grammatical interpretation of the terms like “games of chance” and “games of skill”.  
“One must always be able to say, instead of 'points, straight lines, and planes', 'tables, chairs, and beer mugs’” (David Hilbert)
- Interpretation of skill can be made transparent by mathematical analysis of artificial games.
- Similar to mathematical models of physics mathematics should be restricted to precise logical implications—including implications based on statistical inference.
- This restriction leads to conservative view, i. e. one gets several sufficient conditions for a predomination of skill based on grave perturbations of symmetry. Of course, some cases will remain open.

# Thank you!

Any questions?

*contact and further information:*

*mail@bewersdorff-online.de*

*www.bewersdorff-online.de*  
(including **sheets** and **references** of this talk)



# Appendix



# District Court, Eastern D. NY, 21 Aug 2012

Case 1:11-cr-00414-JBW Document 109 Filed 08/21/12 Page 1 of 120 PageID #: <pageID>

UNITED STATES DISTRICT COURT  
EASTERN DISTRICT OF NEW YORK

UNITED STATES OF AMERICA

– against –

LAWRENCE DICRISTINA,

Defendant.

MEMORANDUM, ORDER,  
& JUDGMENT

11-CR-414

Finally, Dr. Heeb calculated the relative proportions that skill and chance contribute to the outcome of poker. He explained his methodology as follows:

If the proportion of the game attributable to skill is 0%, then the probability of prevailing should be the same for all contestants. As a result, in a matchup of a “higher skilled” and a “lower skilled” player in a game in which skill contributes 0% to the outcome, the “higher skilled” player will win about 50% of the time. If the proportion of skill in the game is 100%, then a perfectly skilled contestant would win 100% of the time. However, even in a game of 100% skill, the more skillful player could make an error, leading to a win by the less skillful player. If skill and chance are equally important, and therefore equally likely to contribute to the probability of prevailing, then half the time skill is decisive, and half of the time chance is decisive. When skill is decisive, the skillful contestant wins.

When chance is decisive, the players have an equal probability of winning. This implies that a contest between two players or teams that is exactly 50% skill and 50% chance will result in the skillful contestant winning 75% of the time. Of course, the poker games I studied were not two player games, but the experiment that I conducted pitted a group of more skilled players against a group of less skilled players, so this is the appropriate measure to use.

# PLOS ONE, March 2, 2015

## Beyond Chance? The Persistence of Performance in Online Poker

Rogier J. D. Potter van Loon<sup>1\*</sup>, Martijn J. van den Assem<sup>2</sup>, Dennie van Dolder<sup>3</sup>

PLOS ONE | DOI:10.1371/journal.pone.0115479 March 2, 2015

A similar approach was used by Randal D. Heeb in his expert report for a U.S. Federal court case in New York in 2012 (Case 1:11-cr-00414-JBW). For one half of the players in his sample, Heeb estimates a regression model that links performance to hundreds of playing style characteristics, including many different variants of *Tightness* and *Aggressiveness* (see previous section). For the other half, he employs the obtained regression coefficients to compute players' predicted performance. Heeb's simulations point out that players who rank high according to this self-constructed skill measure are ahead of lower-ranked players more than 75 percent of the time after only a few hundred hands. A weakness of Heeb's analysis is that he measures players' characteristics and their performance over the same set of hands. This is likely to lead to spurious correlation between skill and performance scores, because both scores are contemporaneously co-determined by the same chance elements. For example, players who are dealt a greater fraction of strong hands or hands that connect well with the community cards are more likely to score high on the dimension of aggressiveness (and thus relatively high on skill) and to record a strong performance. Consequently, his analysis is likely to produce an underestimation of the critical number of hands above which skill predominates.

In our simulations, skill predominates if the more skilled players outperform their less skilled counterparts more than 75 percent of the time. This threshold follows from a simple model where we define the skill factor,  $\pi_h$ , as the probability that skill determines the more profitable player across  $h$  different hands. Accordingly,  $1 - \pi_h$  is the chance factor, or the probability that chance determines the more profitable player. When skill alone determines the winner ( $\pi_h = 1$ ), the more skilled player always wins; when chance alone determines the winner ( $\pi_h = 0$ ), the more skilled player wins half the time. More generally, the overall probability that the more skilled player is ahead after  $h$  hands is equal to  $p_h = \pi_h \cdot 1 + (1 - \pi_h) \cdot 0.5$ . Skill predominates when  $\pi_h > 0.5$ , implying  $p_h > 0.75$ .

# Laustetter, 2012

JR Heft 12/2012 Christian Laustetter Die Abgrenzung des strafbaren Glücksspiels vom straflosen Geschicklichkeitsspiel

## Die Abgrenzung des strafbaren Glücksspiels vom straflosen Geschicklichkeitsspiel<sup>1</sup>

Von Dr. Christian Laustetter, Celle<sup>2</sup>

### VII. Ergebnis

Als Ergebnis lässt sich schlussendlich festhalten, dass nach gegenwärtigem Stand kein Maßstab zur Abgrenzung des Glücksspiels vom Geschicklichkeitsspiel existiert, der für jegliche Spiele eine tragfähige Lösung bieten kann. Zwar existieren in nationaler und internationaler Rechtsprechung und Literatur zahlreiche Ansätze für eine diesbezügliche Einordnung, jedoch kann keiner dieser Abgrenzungsmaßstäbe eine Universallösung des Problems bieten.

Zumindest für Spiele, die gegen andere Spieler und nicht gegen die Bank oder einen Automaten ausgetragen werden und bei denen die Zufallswahrscheinlichkeiten für alle teilnehmenden

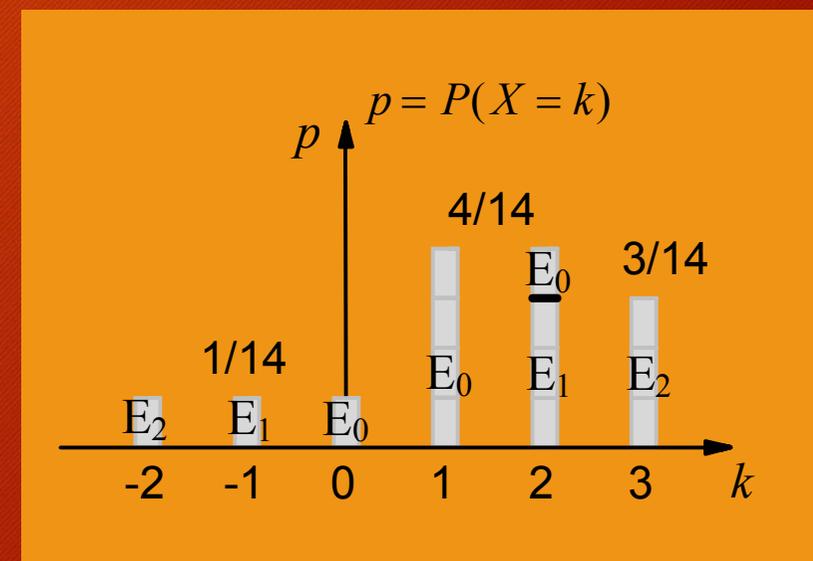
Spieler gleich verteilt sind, kann die Abgrenzung anhand einer praktischen Studie vorgenommen werden, bei der untersucht werden muss, ob ein durchschnittlicher Spieler mit soliden Kenntnissen und Fähigkeiten gegen einen rein nach Zufallsaspekten agierenden Spieler mehr als 75% der durchgeführten Spiele für sich entscheiden kann. Sollte dies der Fall sein, ist das jeweilige Spiel als Geschicklichkeitsspiel einzustufen.

Für die Einordnung des Pokerspiels als Glücksspiel oder Geschicklichkeitsspiel ist dieser Ansatz insofern zu modifizieren, als dass beurteilt werden muss, ob ein durchschnittlicher Spieler nach einer hinreichenden Anzahl an Spielen gegen einen zufällig agierenden Spieler mehr als 75% der im Spiel befindlichen Jetons besitzt.

# A sharper criterion

Given is a symmetric two-person game and the distribution  $X$  of payoffs of a good player against a worse player.

- If  $3 P(X \leq -t) \leq P(X \geq t)$  for all  $t \geq 0$  is fulfilled then the game is predominated of skill.
- For a normal distributed random variable  $X$  it is sufficient that  $E(X) > \Phi^{-1}(\frac{1}{4}) \sigma_X$  if fulfilled.  
The constant is approx. 0.675.



# German National Tournament of Skat

- Results of the tournaments form 2011 to 2016.
- The total number of participants varied form 489 to 763.
- Listed are all players who reached rank 1 to 3 at least once.

Good Players	Year					
	2011	2012	2013	2014	2015	2016
A		0,314			0,936	0,002
B					0,229	0,003
C	0,267	0,707	0,166	0,667	0,777	0,005
D		0,944	0,018		0,001	
E	0,526	0,053	0,303	0,521	0,004	0,497
E				0,001	0,177	
F	0,708			0,003		
G		0,306		0,004		0,676
H			0,002	0,036	0,022	0,039
I		0,579	0,004		0,604	
J	0,149		0,006	0,278		
K		0,001	0,033	0,976	0,804	0,045
L	0,003		0,022	0,192	0,166	0,197
M	0,004		0,184	0,738		

# World Chess Championships

- Results of the World Chess Championships 2008 to 2016,
- differentiated on white and black.

World Chess Championship	White	Black	games with result ...			mean value	standard deviation
			1	0	-1		
2008, Bonn	Anand	Kramnik	5	1	0	0,8333	0,4082
	Kramnik	Anand	1	2	2	-0,2000	0,8367
2010, Sofia	Anand	Topalov	2	4	0	0,3333	0,5164
	Topalov	Anand	2	3	1	0,1667	0,7528
2012, Moscow	Anand	Gelfand	1	5	0	0,1667	0,4082
	Gelfand	Anand	1	5	0	0,1667	0,4082
2013, Chennai	Anand	Carlsen	0	3	2	-0,4000	0,5477
	Carlsen	Anand	1	4	0	0,2000	0,4472
2014, Sochi	Carlsen	Anand	3	3	0	0,5000	0,5477
	Anand	Carlsen	1	4	0	0,2000	0,4472
2016, New York	Carlsen	Karjakin	1	4	1	0,0000	0,6325
	Karjakin	Carlsen	0	6	0	0,0000	0,0000

# Approaches to measure skill in games

- Van der Genugten, Dreef, Borm (2001–2016):  
Skill measure based on game theory (comparing different strategies).
- Fiedler, Rock (2008/2009):  
Comparison of mean value and standard deviation in empirical data.
- Erdmann (2009/2010):  
Skill measure based on game theory and computer simulations.
- Laustetter (2011/2012):  
Legal view, perturbed symmetry of games.
- Bewersdorff (2017/2018):  
Thought experiments with artificial games, perturbed symmetry of games.
- Dürsch, Lambrecht, Oechssler (2017):  
Empirical comparison of real games with 50%-chess, 25%-chess etc.

Without warranty of completeness!